PROBLEM SOLUTIONS

12.1  (a) \[ |F| = kx, \text{ so } k = \frac{|F|}{x} = \frac{230 \text{ N}}{0.4 \text{ m}} = 575 \text{ N/m} \]

(b) \[ U_i = \frac{1}{2} kx^2 = \frac{1}{2}(575 \text{ N/m})(0.4 \text{ m})^2 = 46.0 \text{ J} \]

12.2  (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the motion is periodic.

(b) To determine the period, we use: \[ x = \frac{1}{2} gt^2 \]

The time for the ball to hit the ground is \[ t = \frac{2x}{g} = \frac{2(4.00 \text{ m})}{9.80 \text{ m/s}^2} = 0.909 \text{ s} \]

This equals one-half the period, so \[ T = 2(0.909 \text{ s}) = 1.82 \text{ s} \]

(c) No. The net force acting on the ball is a constant given by \( F = -mg \) (except when it is in contact with the ground), which is not in the form of Hooke's law.

12.3  \( x = (4.00 \text{ m}) \cos(3.00\pi t + \pi) \) Compare this with \( x = A \cos(\omega t + \phi) \) to find

(a) \[ \omega = 2\pi f = 3.00\pi \]

or \[ f = 1.50 \text{ Hz} \quad T = \frac{1}{f} = 0.667 \text{ s} \]

(b) \[ A = 4.00 \text{ m} \]

(c) \[ \phi = \pi \text{ rad} \]

(d) \[ x(t = 0.250 \text{ s}) = (4.00 \text{ m}) \cos(1.75\pi) = 2.83 \text{ m} \]

12.4  (a) \[ 20.0 \text{ cm} \]

(b) \[ v_{\max} = \omega A = 2\pi f A = 94.2 \text{ cm/s} \]

This occurs as the particle passes through equilibrium.

(c) \[ a_{\max} = \omega^2 A = (2\pi f)^2 A = 17.8 \text{ m/s}^2 \]

This occurs at maximum excursion from equilibrium.
12.5  (a) \( x = (5.00 \text{ cm}) \cos \left( 2t + \frac{\pi}{6} \right) \) At \( t = 0 \), \( x = (5.00 \text{ cm}) \cos \left( \frac{\pi}{6} \right) = 4.33 \text{ cm} \)

(b) \( v = \frac{dx}{dt} = -(10.0 \text{ cm/s}) \sin \left( 2t + \frac{\pi}{6} \right) \) At \( t = 0 \), \( v = -5.00 \text{ cm/s} \)

(c) \( a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2) \cos \left( 2t + \frac{\pi}{6} \right) \) At \( t = 0 \), \( a = -17.3 \text{ cm/s}^2 \)

(d) \( A = 5.00 \text{ cm} \) and \( T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = 3.14 \text{ s} \)

\[ k = \left| \frac{F}{x} \right| = \frac{10.0 \times 10^{-3} \text{ kg} \cdot (9.80 \text{ m/s}^2)}{3.90 \times 10^{-3} \text{ m}} = 2.51 \text{ N/m} \quad \text{and} \quad T = 2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{\frac{25.0 \times 10^{-3} \text{ kg}}{2.51 \text{ N/m}}}} = 0.627 \text{ s} \]

12.7  (a) At \( t = 0 \), \( x = 0 \) and \( v \) is positive (to the right). Therefore, this situation corresponds to \( x = A \sin \omega t \).

\[ v = v_0 \cos \omega t \]

Since \( f = 1.50 \text{ Hz} \), \( \omega = 2\pi f = 3.00\pi \)

Also, \( A = 2.00 \text{ cm} \), so that \( x = (2.00 \text{ cm}) \sin 3.00\pi t \)

(b) \( v_{\text{max}} = v_0 = A\omega = 2.00(3.00\pi) = 6.00\pi \text{ cm/s} \)

The particle has this speed at \( t = 0 \) and next at \( t = \frac{T}{2} = \frac{1}{3} \text{ s} \)

(c) \( a_{\text{max}} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2 \text{ cm/s}^2 \)

This positive value of acceleration first occurs at \( t = \frac{3}{4}T = 0.500 \text{ s} \)

(d) Since \( T = \frac{2}{3} \text{ s} \) and \( A = 2.00 \text{ cm} \), the particle will travel 8.00 cm in this time.

Hence, in 1.00 s \( (\frac{3}{4}T) \), the particle will travel 8.00 cm + 4.00 cm = 12.0 cm

*12.8  (a) \( T = \frac{12.0 \text{ s}}{5} = 2.40 \text{ s} \)

(b) \( f = \frac{1}{T} = \frac{1}{2.40} = 0.417 \text{ Hz} \)

(c) \( \omega = 2\pi f = 2\pi(0.417) = 2.62 \text{ rad/s} \)
Chapter 12

12.12 \[ x = A \cos \omega t \quad A = 0.05 \text{ m} \quad v = -A \omega \sin \omega t \quad a = -A \omega^2 \cos \omega t \]

If \( f = 3600 \text{ rev/min} = 60 \text{ Hz} \), then \( \omega = 120 \pi \text{ s}^{-1} \)

\[ v_{\text{max}} = 0.05(120\pi) \text{ m/s} = 18.8 \text{ m/s} \quad a_{\text{max}} = 0.05(120\pi)^2 \text{ m/s}^2 = 7.11 \text{ km/s}^2 \]

12.13 \[ \omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \]

(a) \[ k = \omega^2 m = \frac{4\pi^2 m}{T^2} \]

(b) \[ m' = \frac{k(T')^2}{4\pi^2} = m \left( \frac{T'}{T} \right)^2 \]

12.14 \[ m = 1.00 \text{ kg}, \quad k = 25.0 \text{ N/m}, \quad A = 3.00 \text{ cm}. \quad \text{At } t = 0, \quad x = -3.00 \text{ cm} \]

(a) \[ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00 \text{ rad/s} \]

so that, \[ T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = 1.26 \text{ s} \]

(b) \[ v_{\text{max}} = A\omega = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s}) = 0.150 \text{ m/s} \]

\[ a_{\text{max}} = A\omega^2 = 3.00 \times 10^{-2} \text{ m}(5.00 \text{ rad/s})^2 = 0.750 \text{ m/s}^2 \]

(c) Because \( x = -3.00 \text{ cm} \) and \( v = 0 \) at \( t = 0 \), the required solution is \( x = -A \cos \omega t \)

or \[ x = -3.00 \cos(5.00t) \text{ cm} \]

\[ v = \frac{dx}{dt} = 15.0 \sin(5.00t) \text{ cm/s} \]

\[ a = \frac{dv}{dt} = 75.0 \cos(5.00t) \text{ cm/s}^2 \]

12.15 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

\[ \frac{1}{2} mv^2 = \frac{1}{2} kx^2 : \quad v = x \sqrt{\frac{k}{m}} = (3.16 \times 10^{-2} \text{ m}) \sqrt{\frac{5.00 \times 10^6}{10^3}} = 2.23 \text{ m/s} \]

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