Chapter 14

PROBLEM SOLUTIONS

*14.1*  
\[ y = y_1 + y_2 = 3.00 \cos (4.00x - 1.60t) + 4.00 \sin (5.00x - 2.00t) \]  evaluated at the given x values.

(a) \( x = 1.00, \ t = 1.00 \)  
\[ y = 3.00 \cos (2.40 \text{ rad}) + 4.00 \sin (+3.00 \text{ rad}) = -1.65 \text{ cm} \]

(b) \( x = 1.00, \ t = 0.500 \)  
\[ y = 3.00 \cos (+3.20 \text{ rad}) + 4.00 \sin (+4.00 \text{ rad}) = -6.02 \text{ cm} \]

(c) \( x = 0.500, \ t = 0 \)  
\[ y = 3.00 \cos (+2.00 \text{ rad}) + 4.00 \sin (+2.50 \text{ rad}) = 1.15 \text{ cm} \]

14.2

14.3

(a) \( y_1 = f(x - vt) \), so wave 1 travels in the \( +x \) direction

(b) \( y_2 = f(x + vt) \), so wave 2 travels in the \( -x \) direction

(b) To cancel, \( y_1 + y_2 = 0: \)
\[
\frac{5}{(3x - 4t)^2 + 2} = \frac{+5}{(3x + 4t - 6)^2 + 2}
\]

\[
(3x - 4t)^2 = (3x + 4t - 6)^2
\]

\[
3x - 4t = \pm (3x + 4t - 6)
\]

for the positive root, \( 8t = 6 \)  
\( t = 0.750 \text{ s} \)

(at \( t = 0.750 \text{ s} \), the waves cancel everywhere)

(c) for the negative root, \( 6x = 6 \)  
\( x = 1.00 \text{ m} \)

(at \( x = 1.00 \text{ m} \), the waves cancel always)
Suppose the waves are sinusoidal.

The sum is

\[ (4.00 \text{ cm}) \sin(kx - \omega t) + (4.00 \text{ cm}) \sin(kx - \omega t + 90.0^\circ) \]

\[ 2(4.00 \text{ cm}) \sin(kx - \omega t + 45.0^\circ) \cos 45.0^\circ \]

So the amplitude is \( 8.00 \text{ cm} \cos 45.0^\circ = 5.66 \text{ cm} \)

14.5

The resultant wave function has the form

\[ y = 2A_0 \cos \left( \frac{\phi}{2} \right) \sin \left( kx - \omega t + \frac{\phi}{2} \right) \]

(a) \[ A = 2A_0 \cos \left( \frac{\phi}{2} \right) = 2(5.00) \cos \left( \frac{-\pi/4}{2} \right) = 9.24 \text{ m} \]

(b) \[ f = \frac{\omega}{2\pi} = \frac{1200\pi}{2\pi} = 600 \text{ Hz} \]

14.6

\[ 2A_0 \cos \left( \frac{\phi}{2} \right) = A_0 \]

so \[ \frac{\phi}{2} = \cos^{-1}\left( \frac{1}{2} \right) = 60.0^\circ = \frac{\pi}{3} \]

Thus, the phase difference is \[ \phi = 120^\circ = \frac{2\pi}{3} \]

This phase difference results if the time delay is \[ \frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v} \]

Time delay = \[ \frac{3.00 \text{ m}}{3(2.00 \text{ m/s})} = 0.500 \text{ s} \]

14.7

Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

\[ \Delta r = 66.0 \text{ m} - 28.0 \text{ m} = 38.0 \text{ m} \]

Since \[ \lambda = \frac{v}{f} \]

Then \[ \frac{\Delta r}{\lambda} = \frac{(\Delta r)/f}{\lambda} = \frac{(38.0 \text{ m})(246 \text{ Hz})}{343 \text{ m/s}} = 27.254 \]

or, \[ \Delta r = 27.254\lambda \]

The phase difference between the two reflected waves is then \[ \phi = 0.254(1 \text{ cycle}) = 0.254(2\pi \text{ rad}) = 91.3^\circ \]
(a) $\Delta r = \sqrt{9.00 + 4.00 - 3.00} = \sqrt{10} - 3.00 = 0.606$ m

The wavelength is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$$

Thus,

$$\frac{\Delta r}{\lambda} = \frac{0.606}{1.14} = 0.530$$

of a wave,

or

$$\Delta \phi = 2\pi(0.530) = 3.33 \text{ rad}$$

(b) For destructive interference, we want

$$\frac{\Delta x}{\lambda} = \frac{0.500}{f} = \frac{\Delta x}{\nu}$$

where $\Delta x$ is a constant in this set up.

$$f = \frac{\nu}{2\Delta x} = \frac{343}{2(0.606)} = 283 \text{ Hz}$$

14.9

Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when

$$\Delta r = (2n-1)\left(\frac{\lambda}{2}\right) \quad \text{with} \quad n = 1, 2, 3, \ldots$$

Then,

$$\sqrt{L^2 + d^2} - L = (n - 1/2)(\nu / f)$$

$$\sqrt{L^2 + d^2} = (n - 1/2)(\nu / f) + L$$

$$L^2 + d^2 = (n - 1/2)^2 (\nu / f)^2 + 2(n - 1/2)(\nu / f)L + L^2$$

$$d^2 - \left(n - \frac{1}{2}\right)^2 (\nu / f)^2 = 2(n - 1/2)(\nu / f)L$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference $\Delta r$ starts from nearly zero when the man is very far away and increases to $d$ when $L = 0$.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to $d \geq (n - 1/2)(\nu / f)$, or

$$\text{number of minima heard} = n_{\text{max}} = \text{greatest integer} \leq d(\nu / f) + 1/2$$

(b) From equation 1, the distances at which minima occur are given by

$$L_n = \frac{d^2 - (n - 1/2)^2 (\nu / f)^2}{2(n - 1/2)(\nu / f)} \quad \text{where} \quad n = 1, 2, \ldots, n_{\text{max}}$$
14.10 \[ y = (1.50 \text{ m}) \sin(0.400x) \cos(200t) = 2A_0 \sin kx \cos \omega t \]

Therefore, \[ k = \frac{2\pi}{\lambda} = 0.400 \text{ rad/m} \quad \lambda = \frac{2\pi}{0.400 \text{ rad/m}} = 15.7 \text{ m} \]

and \[ \omega = 2\pi f \quad \text{so} \quad f = \frac{\omega}{2\pi} = \frac{200 \text{ rad/s}}{2\pi \text{ rad}} = 31.8 \text{ Hz} \]

The speed of waves in the medium is \[ v = \lambda f = \frac{\lambda}{2\pi f} \quad \omega = \frac{200 \text{ rad/s}}{0.400 \text{ rad/m}} = 500 \text{ m/s} \]

14.11 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being \[ d_{NN} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(800 \text{ s}^{-1})} = 0.214 \text{ m} \]

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of \[ \frac{1.25 \text{ m}}{2} = 0.625 \text{ m} \]

Then there is a node at \[ 0.625 - \frac{0.214}{2} = 0.518 \text{ m} \]

a node at \[ 0.518 - 0.214 \text{ m} = 0.303 \text{ m} \]

a node at \[ 0.303 - 0.214 \text{ m} = 0.0891 \text{ m} \]

a node at \[ 0.518 + 0.214 \text{ m} = 0.732 \text{ m} \]

a node at \[ 0.732 + 0.214 \text{ m} = 0.947 \text{ m} \]

and a node at \[ -0.947 + 0.214 \text{ m} = 1.16 \text{ m} \] from either speaker

14.12 (a) The resultant wave is \[ y = 2A \sin \left( kx + \phi \right) \cos(\omega t - \phi) \]

The nodes are located at \[ kx + \frac{\phi}{2} = n\pi \]

so \[ x = \frac{n\pi}{k} - \frac{\phi}{2k} \]

which means that each node is shifted \[ \phi / 2k \] to the left.

(b) The separation of nodes is \[ \Delta x = \left[ \frac{(n + 1)\pi}{k} - \frac{\phi}{2k} \right] - \left[ \frac{n\pi}{k} + \frac{\phi}{2k} \right] = \frac{\pi - \lambda}{k} \]

The nodes are still separated by half a wavelength.

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