Momentum and Conservation of Momentum

- Momentum
- Conservation of Momentum
- Impulse-Momentum Theorem
- Homework
Momentum

- The linear momentum, \( p \), of a particle with mass, \( m \), moving with a velocity, \( v \), is defined as

\[
p = mv
\]

- In general, the momentum, \( p \), has three components

\[
p_x = mv_x \quad p_y = mv_y \quad p_z = mv_z
\]

- Alternative statement of Newton’s 2\(^{nd}\) law: The time rate of change of momentum of a particle is equal to the net force acting on the particle.

\[
\sum F = \frac{dp}{dt}
\]

- If the mass of the particle is constant, we get our previous expression for Newton’s 2\(^{nd}\) law

\[
\sum F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma
\]
Conservation of Momentum

- The total momentum for a system of particles is
  \[ p_{tot} = \sum p_i = \sum m_i v_i \]

- Applying Newton’s 2nd law, we have
  \[ \sum F_{ext} = \frac{dp_{tot}}{dt} \]

- If \( \sum F_{ext} = 0 \), then \( \frac{dp_{tot}}{dt} = 0 \) so that
  \[ p_{tot} = \text{constant} \]
Example 1

A bullet with a mass of 3.8 g is fired horizontally with a speed of 1100 m/s into a large block of wood with a mass of 12 kg resting on a frictionless surface. What is the speed of the block after impact?
Example 1 Solution

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\[
\begin{align*}
\text{Initial} & \quad \text{Final} \\
\begin{array}{c}
\text{m} \\
\vec{v}_i
\end{array} & \quad \begin{array}{c}
\text{M} \\
\text{M}+\text{m}
\end{array} & \quad \begin{array}{c}
\vec{v}_f
\end{array}
\end{align*}
\]

\[
\mathbf{p}_i = \mathbf{p}_f
\]
\[
mv_i = (M + m)v_f
\]
\[
v_f = \frac{m}{M + m}v_i
\]

\[
v_f = \frac{3.8 \times 10^{-3} \text{kg}}{12 \text{ kg} + 3.8 \times 10^{-3} \text{kg}} (1100 \text{ m/s}) = 0.35 \text{ m/s}
\]
Impulse-Momentum Theorem

- Assume that a net force $\sum F$ acts on a particle and that it may vary with time as shown below.

- We can use Newton’s 2\textsuperscript{nd} law to write the differential change in momentum, $d\mathbf{p}$, in the differential element of time, $dt$, as

$$d\mathbf{p} = \sum F \, dt$$
Impulse-Momentum Theorem (cont’d)

We can integrate this expression to find the change in momentum, $\Delta p$, of the particle during the time interval $\Delta t = t_f - t_i$

$$\int_{p_i}^{p_f} dp = \int_{t_i}^{t_f} \Sigma F \, dt$$

$$\Delta p = p_f - p_i = \int_{t_i}^{t_f} \Sigma F \, dt$$

The integral of force over the time interval during which it acts is a vector quantity called the impulse, $I$, defined by

$$I \equiv \int_{t_i}^{t_f} \Sigma F \, dt = \Delta p$$

We can also express the impulse in terms of a time-averaged net force $\Sigma \overline{F}$

$$I = \Delta p = \Sigma \overline{F} \Delta t$$
Example 2

A 0.14-kg baseball, in horizontal flight with a speed $v_i$ of 39 m/s, is struck by a batter. After leaving the bat, the ball travels in the opposite direction with a speed $v_f$, also 39 m/s. (a) What impulse acted on the ball while it was in contact with the bat? (b) What average force acts on the baseball if the impact time is 1.2 ms?
Example 2 Solution

A 0.14-kg baseball, in horizontal flight with a speed $v_i$ of 39 m/s, is struck by a batter. After leaving the bat, the ball travels in the opposite direction with a speed $v_f$, also 39 m/s. (a) What impulse acted on the ball while it was in contact with the bat? (b) What average force acts on the baseball if the impact time is 1.2 ms?

(a) \[ I = p_f - p_i = mv_f \mathbf{i} - (-mv_f \mathbf{i}) = m(v_f + v_i) \mathbf{i} \]
\[
I = (0.14 \text{ kg})(39 \text{ m/s} + 39 \text{ m/s}) \mathbf{i} = 11 \text{ kg} \cdot \text{m/s} \mathbf{i}
\]

(b) \[ F = \frac{I}{\Delta t} = \frac{11 \text{ kg} \cdot \text{m/s} \mathbf{i}}{0.0012 \text{ s}} = 9200 \text{ N} \mathbf{i}
\]
Homework Set 14 - Due Wed. Oct. 13

- Read Sections 8.1-8.2
- Answer Questions 8.3 & 8.9
- Do Problems 8.1, 8.4, 8.7 & 8.10