Rotational Dynamics I

- Rotational Kinetic Energy
- Moment of Inertia
- Calculating the Moment of Inertia
- Vector Product
- Torque
- Newton’s 2nd Law for Rotation
- Homework
Rotational Kinetic Energy

\[ K_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2 \]

\[ K = \frac{1}{2} \left( m_1 r_1^2 + m_2 r_2^2 + \ldots \right) \omega^2 \]

\[ K = \frac{1}{2} \left( \sum_i m_i r_i^2 \right) \omega^2 \]

\[ K = \frac{1}{2} I \omega^2 \]

\[ I = \sum_i m_i r_i^2 \quad \Rightarrow \quad \text{Moment of Inertia} \]
Calculating the Moment of Inertia

- **Discrete mass distribution**

\[ I = \sum_i m_i r_i^2 \]

- **Continuous mass distribution**

\[ I = \lim_{\Delta m_i \to 0} \sum_i \Delta m_i r_i^2 = \int r^2 dm \]
Example 1

Consider a rigid body consisting of two particles of mass $M$ connected by a rod of length $L$ and negligible mass. (a) What is the rotational inertia about an axis through its center at right angles to the rod? (b) What will be the rotational kinetic energy of the rigid body if it rotates about the axis with an angular velocity $\omega$?
Example 1 Solution

Consider a rigid body consisting of two particles of mass $M$ connected by a rod of length $L$ and negligible mass. 

(a) What is the rotational inertia about an axis through its center at right angles to the rod?

$$I = \sum m_i r_i^2 = M \left( \frac{L}{2} \right)^2 + M \left( \frac{L}{2} \right)^2 = \frac{1}{2} M L^2$$

(b) What will be the rotational kinetic energy of the rigid body if it rotates about the axis with an angular velocity $\omega$?

$$K = \frac{1}{2} I \omega^2 = \frac{1}{4} M L^2 \omega^2$$
<table>
<thead>
<tr>
<th>Geometry</th>
<th>Moment of Inertia Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoop or thin cylindrical shell</td>
<td>$I_{CM} = MR^2$</td>
</tr>
<tr>
<td>Hollow cylinder</td>
<td>$I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$</td>
</tr>
<tr>
<td>Solid cylinder or disk</td>
<td>$I_{CM} = \frac{1}{2} MR^2$</td>
</tr>
<tr>
<td>Rectangular plate</td>
<td>$I_{CM} = \frac{1}{12} M(a^2 + b^2)$</td>
</tr>
<tr>
<td>Long thin rod with rotation axis through center</td>
<td>$I_{CM} = \frac{1}{12} ML^2$</td>
</tr>
<tr>
<td>Long thin rod with rotation axis through end</td>
<td>$I = \frac{1}{3} ML^2$</td>
</tr>
<tr>
<td>Solid sphere</td>
<td>$I_{CM} = \frac{2}{5} MR^2$</td>
</tr>
<tr>
<td>Thin spherical shell</td>
<td>$I_{CM} = \frac{2}{3} MR^2$</td>
</tr>
</tbody>
</table>
The Vector (or Cross) Product

\[ \mathbf{C} = \mathbf{A} \times \mathbf{B} \]

\[ C = |\mathbf{C}| = AB \sin \theta \]

- The direction of the vector \( \mathbf{C} \) is given by the right-hand rule.
- The vector product is not commutative

\[ \mathbf{A} \times \mathbf{B} = - (\mathbf{B} \times \mathbf{A}) \]
Example 2

Consider the two vectors \( \mathbf{A} = 3\mathbf{i} + 5\mathbf{j} \) and \( \mathbf{B} = 2\mathbf{i} + 4\mathbf{j} \).
Find \( \mathbf{A} \times \mathbf{B} \).
Example 2 Solution

Consider the two vectors $A = 3i + 5j$ and $B = 2i + 4j$. Find $A \times B$.

$$A \times B = (3i + 5j) \times (2i + 4j)$$

$$A \times B = (3)(2)(i \times i) + (3)(4)(i \times j) + (5)(2)(j \times i) + (5)(4)(j \times j)$$

$$A \times B = 0 + (3)(4)k + (5)(2)(-k) + 0$$

$$A \times B = 12k - 10k = 2k$$
Torque

\[ \tau = r \times F \]

\[ \tau = rF \sin \phi = F (r \sin \phi) = Fd \]

\[ d = r \sin \phi \quad \Rightarrow \quad \text{Moment Arm} \]
Newton’s 2nd Law for Rotation

\[ \tau = F_t r = m a_t r = m (\alpha r) r = (mr^2) \alpha \]

\[ \tau = I \alpha \]

\[ \sum \tau = I \alpha \]
Example 3

Consider a uniform disk with a mass 2.5 kg and radius 0.20 m mounted on a fixed horizontal axis. A block with a mass of 1.2 kg hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord.
Example 3 Solution

Consider a uniform disk with a mass 2.5 kg and radius 0.20 m mounted on a fixed horizontal axis. A block with a mass of 1.2 kg hangs from a light cord that is wrapped around the rim of the disk. Find the acceleration of the falling block, the angular acceleration of the disk, and the tension in the cord.

$$\sum \tau = I \alpha$$

$$RT = \left(\frac{1}{2}MR^2\right) \left(\frac{a}{R}\right)$$

$$T = \frac{1}{2}Ma$$
Example 3 Solution (cont’d)

\[ \sum F_y = T - mg = -ma \]
\[ \frac{1}{2} Ma - mg = -ma \]
\[ a = \frac{2m}{M + 2m} g \]

\[ a = \frac{2(1.2 \text{ kg})}{2.5 \text{ kg} + 2(1.2 \text{ kg})} (9.8 \text{ m/s}^2) = 4.8 \text{ m/s}^2 \]

\[ T = \frac{1}{2} Ma = \frac{1}{2} (2.5 \text{ kg})(4.8 \text{ m/s}^2) = 6.0 \text{ N} \]

\[ \alpha = \frac{a}{R} = \frac{4.9 \text{ m/s}^2}{0.20 \text{ m}} = 24 \text{ rad/s}^2 \]
Homework Set 19 - Due Wed. Oct. 27

- Read Sections 10.4-10.5 & 10.7
- Answer Question 10.15
- Do Problems 10.13, 10.15, 10.17 & 10.20