Rotational Kinematics

- Angular Position
- Angular Velocity
- Angular Acceleration
- Rotation with Constant Angular Acceleration
- Homework
Angular Position

- Consider an object rotating about a fixed axis through $O$ perpendicular to the plane as shown below.
- A particle at point $P$ has an angular position

$$\theta = \frac{s}{r}$$

where $s$ is the linear distance through which the point moves and $r$ is the distance of the point $P$ from the axis.
- The unit of angular position is the radian (rad).
- 1 rev. $= 360^\circ = 2\pi$ rad
Angular Velocity

- **Average angular velocity**

  \[
  \bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta \theta}{\Delta t}
  \]

- **Instantaneous angular velocity**

  \[
  \omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}
  \]
Relation Between Angular and Linear Speed

\[ s = \theta r \]
\[ \frac{ds}{dt} = \frac{d\theta}{dt} \]
\[ v = \omega r \]
Angular Acceleration

- Average angular acceleration

\[ \alpha = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta \omega}{\Delta t} \]

- Instantaneous angular acceleration

\[ \alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \]
Relation Between Angular and Linear Acceleration

\[ v = \omega r \]
\[ \frac{dv}{dt} = \frac{d\omega}{dt} r \]
\[ a_t = \alpha r \]

\[ a_r = \frac{v^2}{r} = \omega^2 r \]
Direction of Angular Velocity Vector

- The direction of the angular velocity vector is given by the right hand rule.
- The four fingers of the right hand are wrapped in the direction of the rotation.
- The extended right thumb points in the direction of $\omega$. 
# Rotation with Constant Angular Acceleration

## Table 10.1

A Comparison of Equations for Rotational and Translational Motion: Kinematic Equations

<table>
<thead>
<tr>
<th>Rotational Motion About a Fixed Axis with ( \alpha = \text{Constant} ) (Variables: ( \theta_f ) and ( \omega_f ))</th>
<th>Translational Motion with ( \alpha = \text{Constant} ) (Variables: ( x_f ) and ( v_f ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_f = \omega_i + \alpha t )</td>
<td>( v_f = v_i + \alpha t )</td>
</tr>
<tr>
<td>( \theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 )</td>
<td>( x_f = x_i + v_i t + \frac{1}{2} \alpha t^2 )</td>
</tr>
<tr>
<td>( \theta_f = \theta_i + \frac{1}{2} (\omega_i + \omega_f) t )</td>
<td>( x_f = x_i + \frac{1}{2} (v_i + v_f) t )</td>
</tr>
<tr>
<td>( \omega_f^2 = \omega_i^2 + 2 \alpha (\theta_f - \theta_i) )</td>
<td>( v_f^2 = v_i^2 + 2 \alpha (x_f - x_i) )</td>
</tr>
</tbody>
</table>
Example

A grindstone wheel has a constant angular acceleration of 0.35 rad/s². (a) If it starts from rest, what is the angular displacement after 18 s? (b) What is the angular speed of the wheel at t = 18 s?
A grindstone wheel has a constant angular acceleration of \(0.35 \text{ rad/s}^2\). (a) If it starts from rest, what is the angular displacement after 18 s?

\[
\omega_i = 0 \quad \alpha = 0.35 \text{ rad/s}^2 \quad t = 18 \text{ s}
\]

\[
\theta_f - \theta_i = \omega_i t + \frac{1}{2} \alpha t^2
\]

\[
\theta_f - \theta_i = 0 + \frac{1}{2} (0.35 \text{ rad/s}^2) (18 \text{ s})^2 = 57 \text{ rad}
\]

\[
\theta_f - \theta_i = 57 \text{ rad} \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) \approx 9 \text{ rev}
\]

(b) What is the angular speed of the wheel at \(t = 18 \text{ s}\)?

\[
\omega_f = \omega_i + \alpha t
\]

\[
\omega_f = 0 + (0.35 \text{ rad/s}^2) (18 \text{ s}) = 6.3 \text{ rad/s}
\]
Homework Set 18 - Due Mon. Oct. 25

- Read Sections 10.1-10.3
- Answer Questions 10.2 & 10.3
- Do Problems 10.1, 10.4, 10.6, 10.9 & 10.12