Work

- Work Done by a Constant Force
- The Scalar (or Dot) Product of Two Vectors
- Work Done by a Variable Force
- Homework
Work Done by a Constant Force

\[ W = F \Delta r \cos \theta \]

The unit of work is the joule (J) (1 J = 1 N\cdot m)
Forces Perpendicular to the Motion Do No Work

When an object is displaced horizontally on a flat table, the normal force \( n \) and the gravitational force \( F_g \) do no work since \( \cos \theta = 90^\circ = 0 \)
The Scalar (or Dot) Product of Two Vectors

The scalar product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$$

where $\theta$ is the angle between $\mathbf{A}$ and $\mathbf{B}$.
Properties of the Scalar Product

- The scalar product is commutative
  \[ \mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} \]

- The scalar product obeys the distributive law
  \[ \mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C} \]

- The scalar product of the unit vectors gives us
  \[ \mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1 \]
  \[ \mathbf{i} \cdot \mathbf{j} = \mathbf{i} \cdot \mathbf{k} = \mathbf{j} \cdot \mathbf{k} = 0 \]

- The scalar product can also be written in terms of the components of \( \mathbf{A} \) and \( \mathbf{B} \) as
  \[ \mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z \]
Work is a Scalar (or Dot) Product

The work done by a constant force is the scalar product of the force and displacement vectors

\[ W = \mathbf{F} \cdot \Delta \mathbf{r} = F \Delta r \cos \theta \]
Work Done by a Variable Force

- Consider a particle being displaced along the x axis under the action of a force of magnitude $F_x$ in the x direction.
- The work done by the force as the particle moves a distance $\Delta x$ is approximately

$$W_1 \approx F_x \Delta x$$
Work Done by a Variable Force (cont’d)

- The total work done as the particle moves from $x_i$ to $x_f$ is approximately

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

- The total work can be found by taking the limit as $\Delta x$ goes to zero

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

- The most general definition of the net work done on a particle is

$$W_{net} = \int_{x_i}^{x_f} (\sum \mathbf{F}) \cdot d\mathbf{r}$$
Work Done by a Constant Force Example

(a) How much work must be done by a force $F$ directed along a 30° frictionless incline to push a block of mass 10 kg up the incline a distance of 5.0 m? (b) How much work would you have to do if you just lifted the block up to the final position?
Work Done by a Constant Force Example

(a) How much work must be done by a force $F$ directed along a $30^\circ$ frictionless incline to push a block of mass 10 kg up the incline a distance of 5.0 m? (b) How much work would you have to do if you just lifted the block up to the final position?
Work Done by a Spring

(a) $F_s$ is negative. $x$ is positive.

(b) $F_s = 0$ $x = 0$

(c) $F_s$ is positive. $x$ is negative.

(d) $\text{Area} = \frac{1}{2} k x_{\text{max}}^2$ $F_s$

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Homework Set 10 - Due Mon. Oct. 4

- Read Sections 6.1-6.4
- Answer Questions 6.2 & 6.4
- Do Problems 6.2, 6.4, 6.7, 6.11 & 6.16